



Performances of Mexican basic-education teachers in solving six matchstick puzzles: Evaluating their difficulties and applicability of these puzzles in classrooms

Arce García Quitzel Yuritzi^a, Slisko Josip^a, Juárez Ruíz Estela de Lourdes^a

^aFacultad de Ciencias Físico Matemáticas BUAP, Av. Universidad & Calle Educación, Ciudad Universitaria, 72592, Heroica Puebla de Zaragoza, Puebla, México.

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E-mail addresses:
yuritzi.arce01@gmail.com,
jslisko@cfm.buap.mx

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ABSTRACT

This article presents an analysis of performances of 67 Mexican basic education teachers in solving five matchstick puzzles. Four puzzles required participants to remove a specific number of matchsticks in the initial configurations to form a determined number of squares. In the fifth puzzle the task was to replace 12 matchsticks in the initial configuration to get a new configuration with minor area. The sixth task was a puzzle-posing one. After seeing a puzzle with Roman numerals, teachers were asked to propose possible new puzzles. All puzzles used in this study have at least two solutions. The analysis was focused mainly on teachers' abilities to find different solutions to the same matchstick puzzle, their perception of the difficulty of the puzzles, and its potential applicability in their respective classrooms. The results show variability in the teachers' solving skills, influenced by the discipline or subject they teach. Teachers from the arts field stood out compared to those teaching other subjects. Moreover, most teachers found the puzzles difficult, citing a lack of time to solve them and missing experiences in solving creativity-based tasks. In order that matchstick puzzles serve as a teaching tool to develop students' mathematical skills, promote their creative thinking, and contribute potentially to students' innovative learning strategies, their number in mathematics textbooks should increase along with a better didactic approach.

Este artículo ofrece un análisis del desempeño de 67 maestros de educación básica en México al resolver cinco acertijos con cerillas. Cuatro de los acertijos requerían que los participantes eliminaran un número específico de cerillas de las configuraciones iniciales para formar una cantidad determinada de cuadrados. En el quinto acertijo, la tarea consistía en reemplazar 12 cerillas de la configuración inicial para obtener una nueva configuración con un área menor. La sexta tarea consistió en que después de observar un acertijo con números romanos, se pidió a los maestros que sugirieran posibles nuevos acertijos. Todos los acertijos utilizados en este estudio tienen al menos dos soluciones. El análisis se centró principalmente en las habilidades de los maestros para encontrar diferentes soluciones al mismo acertijo, su percepción sobre la dificultad de los acertijos y su posible aplicación en las aulas. Los resultados muestran una variabilidad en las habilidades de resolución de los maestros, influida por la disciplina o materia que enseñan. Los maestros del área de artes destacaron en comparación con aquellos que enseñan otras materias. Además, la mayoría de los maestros consideró los acertijos difíciles, mencionando la falta de tiempo para resolverlos y la ausencia de experiencias previas con tareas que fomenten la creatividad. Para que los acertijos con cerillas puedan servir como herramienta educativa para desarrollar habilidades matemáticas en los estudiantes, promover su pensamiento creativo y contribuir potencialmente a estrategias de aprendizaje innovadoras, se sugiere aumentar su presencia en los libros de texto de matemáticas junto con un mejor enfoque didáctico.

I. INTRODUCTION

In the 21st century, the development of problem-solving skills is fundamental in everyday life, as the process of solving problems—such as generating different ideas or applying various methods—is more significant than simply arriving at the correct answer itself (Mayer et al., 1995 cited by Kurbal, 2015). The results of the PISA 2022 indicate that Mexico is 77 points below the average in the area of mathematics (Staff, 2023), a subject that is crucial in basic education, with its practical uses being key to the development of a country (Diah, 2020).

Among the main causes of this issue, Bacelar Valente (2020) states that educators continue to teach mathematics in a routine, algorithmic, and extensive manner, without using innovative methodologies, strategies, or tools that spark interest among students. Additionally, Wahyudi and collaborators (2019) note that teachers do not prioritize updating or innovating their pedagogical knowledge. This problem not only affects teachers but also students, with factors such as motivation for their own learning, the ability to comprehend problems, and visual-spatial skills, all of which may reduce their problem-solving capacity in mathematics (Diah, 2020; Vargas, 2021 cited by Yupanqui Valverde, 2023).

Mumu and collaborators (2017) stated rightly that the teacher is the key agent in building problem-solving skills, and therefore new strategies and/or teaching tools must be sought to improve student learning. Many methods can be used to improve reasoning skills at an early age. However, there is general agreement that reasoning skills can be enhanced through playing games (Kiili, 2007; McFarlane, Sparrowhawk & Heald, 2002 cited by Kurbal, 2015).

Quintanilla (2021) pointed out that games represent a key teaching tool in the development of mathematical skills, allowing children to develop their cognitive independence through specific learning situations that stimulate their activity. This implies the need to use recreational strategies to develop mathematical skills. The implementation of these games has benefited the development of students' skills. For example, in Turkey, the "Puzzles and Games" course was implemented in the academic program in 2013, and students reported that the course helped them improve their problem-solving skills (Kurbal, 2015). Among different kinds of puzzles, perhaps the most popular are matchstick (or toothpick) puzzles.

II. MATCHSTICK PUZZLES: SHORT HISTORIC COMMENT AND DIFFERENT USES

Matchstick puzzles have a very long history. The first one was published in 1849 by the magazine "*The Family Friend*", accompanied by the following description: "Cut 17 paper strips or wood of equal length and place them on the table to form 6 squares as shown in the diagram. Remove 5 strips to leave only 3 squares." The diagram mentioned in the puzzle had the form as in the Figure 1, with matchsticks used instead of the paper strips.



FIGURE 1. The configuration of matchsticks in the puzzle published in 1849.



FIGURE 2. The solution of the puzzle published in 1849.

Although the puzzle has two solutions, only one was published (Figure 2). Even today the authors publish only one solution.

Forty years later, in 1889, Sophus Tromholt (Figure 3) published in Germany the first book, with the title “*Streichholzspiele*” (“*Game with matchsticks*”), that contained more than 250 games and puzzles with matchsticks (Figure 4).



FIGURE 3. Sophus Tromholt (1851 – 1896).



FIGURE 4. The cover page of Tromholt’s book

Although it was never translated into English, many puzzles and games from Tromholt's book were "borrowed" by the authors of the first and actual books written in that language for general readers (Hoffman, 1893; Blyth, 1921; Abraham, 1932; Bakst, 1954; Brooke, 1973; Botermans, 2006).

Today matchstick (or toothpick) puzzles are widely used with different purposes and in different contexts.

In mathematics education, matchstick (or toothpick) puzzles can be found as sporadic funny tasks in textbooks for the students (Altieri *et al.*, 2007; Bornhold *et al.*, 1982; Diniz, 2018; Joshua, 1991; Langbort & Thompson, 1985; Payne *et al.*, 1985) and in the manuals for teachers (Overholt & Kincheloe, 2010; Johnson, 2012).

To that fact should be added the use of matchstick puzzles in a series of books whose authors try to contribute to the development of significant personal abilities for successful work in number of professions, from business managers (Plenert, 1995; Bothe, 2002) to architectural designers (Laseau, 2000; Roberts et al., 2017). It is interesting to mention that solving such puzzles are recommended as one of the useful mental activities that reduce the risk of Alzheimer's disease (Kosik, 2015; Small & Vorgan, 2023).

Especially important are those books in which the matchstick (or toothpick) puzzles are used for instructing the readers how to improve their brain power (Carter & Russel, 2002; Owen, 2009; Harrison & Hobs, 2010; Goldsmith, 2012; Restak, 2013) or boost their creativity (Allen, 2006).

The objective of this qualitative study (Baéz & de Tudela, 2009) was to find initial answers to three research questions:

1. What are Mexican basic-education teachers' performances in solving different matchstick puzzles?
2. How the teachers evaluate difficulties of those puzzles for themselves?
3. How the teachers evaluate difficulties of those puzzles for their students?

III. SAMPLE AND METHODOLOGY FOR THE FIRST ACTIVITY

The sample involved a group of 67 basic-education teachers from private and public primary and secondary schools in Puebla City (Mexico), during a 6-hour long professional-development meeting. At the beginning, teachers are asked to provide information such as age, gender, work experience, current subjects taught, and other subjects they have taught in the past.

The first puzzle-solving activity had four matchstick puzzles in pencil-and-paper form. All puzzles shared the same initial configuration with 17 matchsticks forming 6 equal squares with side-length of 1 matchstick. The solving time was approximately 45 minutes

Instructions indicating that the sequence of puzzles involved progressively removing more matches to create different numbers of squares. It was also specified that the matches to be removed should be marked with an "X" in the image and that the remaining matches must form the required squares.

The instructions for the four puzzles with removing matchsticks were as follows:

Puzzle 1. Remove one match to obtain five squares. There are two solutions (Figure 5).

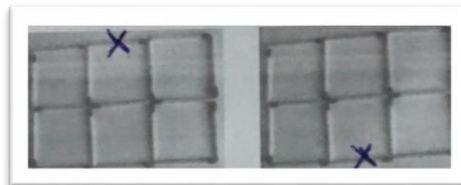


Figure 5. Two solutions of the first puzzle not shown to the teachers.

Puzzle 2. Remove two matches to obtain five squares. There are four solutions (Figure 6).

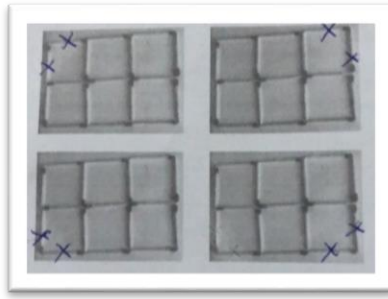


Figure 6. Four solutions of the second puzzle not shown to the teachers.

Puzzle 3. Remove three matches to obtain four squares. There are five solutions (Figure 7).

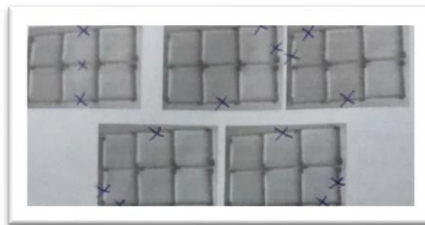


Figure 7. Five solutions of the third puzzle not shown to the teachers.

Puzzle 4. Remove four matches to obtain three squares. There are two solutions (Figure 8).

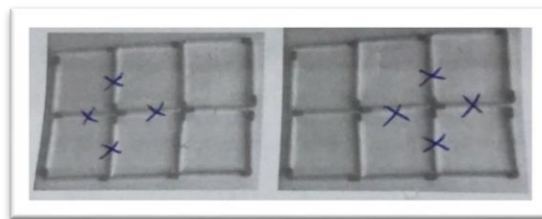


Figure 8. Two solutions of the fourth puzzle not shown to the teachers.

The final part of the test included two questions to assess the participants' perceptions of the puzzles' difficulty and their potential applicability for their students. The questions were:

1. Do you find these matchstick puzzles to be?
2. For your students, would these matchstick puzzles be...?

Each question provided five difficulty levels:

Very easy

Easy

Neither easy nor difficult

Difficult

Very difficult

To choose one and a space for free-response justifications for the evaluation.

Participants were asked to mark their evaluation with a letter “X.”

IV. RESULTS OF PUZZLE - SOLVING WITH MATCHSTICK REMOVING AND EVALUATION ACTIVITY

IV.1 Puzzle solving with matchstick removing

Puzzle 1. Remove one match to obtain five squares. There are two solutions.

The performances of teachers are shown in the following table.

TABLE I. The results of teachers in solving the puzzle 1.

<i>Number of correct solutions</i>	<i>Number of teachers in correspond answer category</i>	<i>Percentage</i>
0	18	26.9 %
1	11	16.4 %
2	36	53.7 %
None response	2	3.0 %

It is seen that 18 teachers couldn't find a single solution while 36 teachers were able to find all possible solutions.

Puzzle 2. Remove two matches to obtain five squares. There are four solutions.

The performances of teachers are shown in the Table II.

TABLE II. The results of teachers in solving the puzzle 2.

<i>Number of correct solutions</i>	<i>Number of teachers in correspond answer category</i>	<i>Percentage</i>
0	7	10.4 %
1	9	13.4 %
2	2	3.0 %
3	3	4.5 %
4	46	68.7 %
None response	0	0 %

It is seen that 7 teachers couldn't find a single solution while 46 teachers were able to find all possible solutions.

Puzzle 3. Remove three matches to obtain four squares. There are five solutions.

The performances of teachers are shown in the Table III.

TABLE III. The results of teachers in solving puzzle 3.

<i>Number of correct solutions</i>	<i>Number of teachers in corresponding answer category</i>	<i>Percentage</i>
0	19	28.4 %
1	30	44.8 %
2	2	3.0 %
3	1	1.5 %
4	1	1.5 %
5	10	15.0 %
None response	4	6.0 %

It is seen that 19 teachers couldn't a single solution while only 10 teachers were able to find all possible solutions.

Puzzle 4. Remove four matches to obtain three squares. There are two solutions.

The performances of teachers are shown in the Table IV.

TABLE IV. The results of teachers in solving the puzzle 4.

<i>Correct solutions</i>	<i>Number of teachers that obtained "n" correct solutions</i>	<i>Percentage</i>
0	29	43.28 %
1	9	13.43 %
2	18	26.86 %
None response	11	16.41 %

It is seen that 29 teachers couldn't single a single solution while 18 teachers were able to find all possible solutions. Comparing teachers' performances in all four puzzles, it turns out that the most difficult was the Puzzle 3 for which only 10 teachers were able to find all 5 solutions.

IV.2 A deeper look at teachers' performances

After taking a deeper look at teachers' performances, the following conclusion can be made:

Teachers who successfully solved the initial puzzles were more likely to attempt subsequent puzzles, indicating a potential correlation between initial success and perseverance.

Teachers' confidence decreased as the puzzles became more challenging, which was reflected in a higher rate of incomplete responses in puzzles 3 and 4.

Teachers who generally scored lower tended to fail in the initial puzzles.

Teachers of arts or humanities performed significantly better, solving all puzzles more frequently and citing creativity as a key factor. Conversely, no mathematics teachers solved all the puzzles correctly, although many achieved more than half.

IV. 3 Puzzles difficulty evaluation: The case of teachers

The evaluation of puzzles difficulty among the teachers varied considerably. The results are shown in the Table V.

TABLE V. Evaluation of puzzles difficulty for teachers themselves

<i>Puzzles difficulty</i>	<i>Number of teachers</i>	<i>Percentage</i>
Very easy	1	1.5 %
Easy	11	16.4 %
Neither easy nor difficult	25	37.3 %
Difficult	24	35.8 %
Very difficult	1	1.5 %
None response	5	7.5 %

The main reasons why the teachers evaluated the difficulty of the puzzles in this way were:

Lack of Skill. They did not feel sufficiently skilled to solve the puzzles.

Limited Time. Participants considered the allotted time insufficient, preventing them from solving all the puzzles.

Lack of Creativity. Some teachers recognized a lack of creativity in finding solutions.

Algorithms or Patterns. Although some participants mentioned identifying an algorithm or pattern, not all were able to apply it correctly.

IV. 4 Puzzles difficulty evaluation: The case of students

When asking to predict puzzles difficulty if they were given their students to solve, the teachers' evaluations were as in the Table VI:

TABLE VI. Difficult evaluation to students

<i>Scale</i>	<i>Number of teachers</i>	<i>Percentage</i>
Very easy	1	1.5 %
Easy	7	10.4 %
Neither easy nor difficult	23	34.3 %
Difficult	17	25.4 %
Very difficult	11	16.4 %

The teachers presented the following arguments on which they based their evaluation:

Resistance to Thinking. A common perception is that students dislike engaging in tasks that require deep thinking.

Low Frustration Tolerance. Teachers noted that students tend to get frustrated easily when facing difficulties.

Impulsivity. Teachers argued that students would respond to the puzzles without taking the time to analyze them.

Higher Creativity. Those teachers, who evaluate the puzzles as “very easy” or “easy”, observed that their students could be more creative and enjoy these activities more than the teachers themselves!

V. RESULTS OF SOLVING TWELVE – TOOTHPICK PUZZLE AND PUZZLE POSING

During the session, the teachers were also asked to solve in groups the twelve-toothpick puzzle:

With twelve matchstick you can build the figure of a cross (Figure 9), whose area is equal to five “toothpick squares”.

Change the arrangement of the matches so that the outline of the figure obtained covers only an area equivalent to four “toothpick squares”. The perimeter of the figure must have length of 12 toothpicks

To solve this puzzle, no measuring instruments of any kind should be used.

One of many possible solutions to the puzzle is shown in the Figure 10.



Figure 9. Initial toothpick configuration for twelve-toothpick puzzle.



Figure 10. One possible solution to twelve-toothpick puzzle.

As it can be seen, the area of the solution figure (with 12-toothpick perimeter) was reduced by one “toothpick square”, transforming the right square into an equilateral triangle and resting from the left square an equilateral triangle.

Teachers were divided in 12 teams and given necessary toothpicks. The solving time was 15 minutes. Each group was supposed to publish the photo of its solution on the wall of Facebook closed group formed for easy sharing solutions and ideas in real time. The solutions of 12 teams are shown in the Figure 11.



Figure 11. Teachers' group solutions to twelve-toothpick puzzle.

Surprisingly, teacher teams were not able to find a correct solution of the twelve-toothpick puzzle! Those solutions with areas of four “toothpick squares” were figures whose perimeter lengths less than 12 toothpicks.

Puzzle posing activity started with puzzling affirmation: “The half of twelve is seven.”

Teachers were asked if they agreed with that affirmation. Fifty comments were recorded on the Facebook wall:

Twenty-six teachers agreed with the affirmation.

Eighteen teachers disagreed with the affirmation.

One teacher was unsure, and five teachers found the affirmation confusing.

Seven participants argued that if the puzzle is approached with Roman numerals, and XII is divided horizontally, the result is VII, thus making the affirmation true.

A teacher presented this argument visually (Figure 12), which was later accepted by all teaches as the correct puzzle solution.

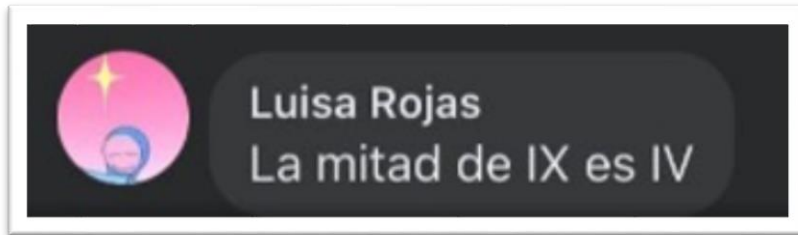


Figure 12. The correct solution presented visually by a teacher.

After the correct solution of the puzzling affirmation was seen, the teachers were asked to propose new puzzling affirmations. Although some of proposed puzzling affirmations revealed a lack of basic Roman numerals knowledge (Figure 13), the performances of teachers were rather good.



Figure 13. The half of 22 is 12.

Generally, teachers posed only one new puzzling affirmation (Figure 14).



Figure 14. The half of IX is IV.

Nevertheless, some teachers were able to pose two solutions (Figures 15 and 16) or even four solutions, one of which was not a puzzling affirmation (Figure 17).

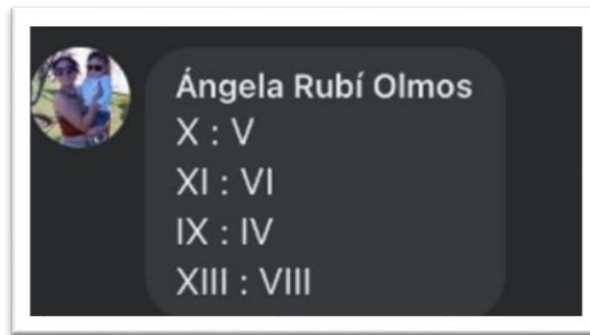


Figure 15. The half of XI is 6 and the half of IX is 4.

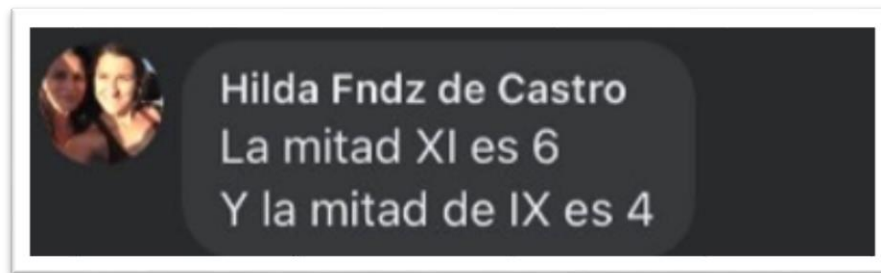


Figure 16. The half of 9 is 4 and the half of 11 is 6.



Figure 17. Three correct puzzling affirmations (the half of XI is VI, the half of IX is IV, the half of XIII is VIII), and one trivial affirmation (the half of X is V).

VI. CONCLUTIONS

The evaluative and exploratory approach of this study not only allows for measuring teachers' current ability to solve specific matchstick and toothpick puzzles but also provides an understanding of their perceptions and opinions about the applicability of these puzzles in the classroom.

This analysis reveals significant variability in problem-solving abilities among teachers, influenced by the subject they teach and their perception of creativity. The solving difficulty of the puzzles increased with their complexity, being maximal in the twelve-toothpick puzzle, affecting the confidence and performance of the teachers.

Additionally, opinions about students' puzzle-solving skills indicate that, although there are significant challenges, some teachers recognize their students' creative potential to enjoy and benefit from this type of activity.

It is important to emphasize that, despite teachers claiming not to have creativity for solving puzzles, they demonstrated creativity when posing similar puzzles, after knowing the solution of puzzling affirmation “the half of twelve is seven”. Therefore, it is expected that some of mathematics teachers, who participated in these professional development activities, will apply some of these puzzles in their respective classrooms to foster meaningful learning for their students.

It is important to stress that, in order that matchstick or toothpick puzzles serve as a teaching tool to develop students' mathematical skills, promote their creative thinking, and contribute potentially to students' innovative learning strategies, their number in mathematics textbooks should increase along with a better didactic approach.

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